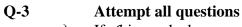
1	Enrollme	nt No:		Exam Seat No:		
-				UNIVERSITY		
		Su	mmer Exa	amination-2020		
\$	Subject N	lame: Linear Algel	bra			
\$	Subject Code: 5SC01LIA1			Branch: M.Sc.(Mathematics)		
S	Semester	: 1 Date :	24/02/2020	Time: 02:30 To 05:30 M	(arks : 70	
<u>]</u>	(2) In (3) D	se of Programmablestructions written o	on main answer be and figures (if ne	any other electronic instrument is prohibit ook are strictly to be obeyed. cessary) at right places.	ed.	
0.4				TION – I	(O=)	
Q-1		Attempt the Follov	.		(07)	
	f	finite dimensional a	nd dim(A + B) =	spaces of a vector space V , then $A + B$ is $= \dim A + \dim B - \dim(A \cap B)$.		
		Prove that $L(S)$ is so			(02)	
				space over F and $S, T \in A(V)$. show that	(02)	
		rank(ST) ≤ rank(Define : Minimal Po	. ,		(01)	
Q-2	I	Attempt all questic	ons		(14)	
			-	prove that V is isomorphic to a subspace of	of (07)	
		$\widehat{\hat{V}}$. If V is finite dime				
				space over F and W be subspace of V . Sh $\leq \dim V$ and $\dim V/W = \dim V - \dim V$		
Q-2		Attempt all question	nns	OR	(14)	
~ ~	a) l		tor space over F	of dimension m and n respectively. Then for mn over F .	, ,	
	-	Prove that $W^{00} = V$	•		(05)	
	c) 1	If $v_1, v_2, \ldots v_n$	are in V then eit	her they are linearly independent or some one's $v_1, v_2, \dots v_{k-1}$.	` ′	



a (14) a (05)

a) If \mathcal{A} is an algebra over F with unit element then prove that \mathcal{A} is isomorphic to a subalgebra of A(V) for some vector space V over F.

b) If V is finite dimensional over F, then prove that $T \in A(V)$ is regular if and only if T maps V on to V.

c) Let V be finite dimensional over F and $T \in A(V)$ show that the number of characteristic root of T is at most n^2 .

(04)



Q-3		Attempt all questions	(14)	
	a)	If V is finite dimensional over F, then prove that $T \in A(V)$ is invertible if and	(05)	
		only if the constant term in the minimal polynomial for T is nonzero.		
	b)	If V is finite dimensional over F, and let $S, T \in A(V)$ and S be regular, then	(05)	
		prove that $\lambda \in F$ is chatracteristic root of T if and only if it is a characteristic root		
	a)	of $S^{-1}TS$.	(04)	
	c)	Let V be a finite dimensional vector space over F. If $T \in A(V)$ is right invertible then T is invertible.	(04)	
		then I is invertible.		
		SECTION – II		
Q-4		Attempt the Following questions	(07)	
	a)	Let $A, B \in M_n(F)$, show that $AB - BA \neq I$.	(02)	
	b)	If $A \in M_n(F)$ is regular then $\det(A) \neq 0$ and $\det(A^{-1}) = \frac{1}{\det A}$.	(02)	
	c)	Find the inertia of quadratic equation $2x_1x_2 + 2x_1x_3 = 0$.	(02)	
	d)	Define: Basic Jordan block.	(01)	
	/		(- /	
Q-5		Attempt all questions	(14)	
	a)	Let V be a finite dimensional vector space over F and $T \in A(V)$ be nilpotent.	(07)	
		Then show that the invariants of T are unique.	(O.T.)	
	b)	Let V be a finite dimensional vector space over F and $T \in A(V)$. If all the	(07)	
		characteristic roots of T are in F then there is a basis of with respect to which the		
		matrix of T is (upper) triangular. OR		
Q-5		Attempt all questions	(14)	
Q U	a)	Let V be a finite dimensional vector space over F and $T \in A(V)$ and W be	(07)	
	,	subspace of V invariant under . Then T induce a map $\overline{T}: V/W \to V/W$ defined	` /	
		by $\overline{T}(v+W) = Tv + W$ show that $\overline{T} \in A(V/W)$. Further \overline{T} satisfies every		
		polynomial satisfies by T. If $p_1(x)$ and $p(x)$ are minimal polynomial for \overline{T} and T		
		respectively then show that $p_1(x)/p(x)$.		
	b)	Two nilpotent linear transformations are similar if and only if they have the same	(07)	
		invariants.		
Q-6		Attempt all questions	(14)	
Q o	a)	Let $A, B \in M_n(F)$, show that $\det(AB) = \det A \cdot \det B$.	(05)	
	b)	Identify the surface given by $11x^2 + 6xy + 19y^2 = 80$. Also convert it to the	(05)	
	ŕ	standard form by finding the orthogonal matrix P .	` ′	
	c)	Interchanging two rows of matrix changes the sign of its determinant.	(04)	
		OR		
Q-6		Attempt all Questions	(14)	
	a)	State and prove Cramer's rule.	(05)	
	b)	Let $f: \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ be a map. Then f is bilinear if and only if there exist	(05)	
	<i>a</i>)	$\alpha_{ij} \in \mathbf{R}, 1 \le i, j \le n$ with $\alpha_{ij} = \alpha_{ji}$ such that $f(x, y) = \sum_{i,j=1}^{n} \alpha_{ij} x_i y_j$.	(0.4)	
	c)	Let F be a field of characteristic 0 and V be a vector space over F. If $S, T \in A(V)$	(04)	
		such that $ST - TS$ commutes with S then show that $ST - TS$ is nilpotent.		

